

Online Appendix

Does Openness Amplify Money-Financed Fiscal Stimulus? A Fiscal Theory Perspective

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Appendix A: Non-policy Block

This appendix presents the full specification of the non-policy block, which follows the standard New Keynesian small open economy framework of Galí and Monacelli (2005, 2016) and Okano and Eguchi (2024).

A.1 Households

The small open economy has a representative household with a continuum of members indexed by $j \in [0, 1]$.

The household utility function is

$$\sum_{t=0}^{\infty} \beta^t [\mathcal{U}(C_t, L_t, N_t; Z_t)], \quad (\text{A.1})$$

where $C_t \equiv \frac{1}{(1-\nu)^{1-\nu}\nu^\nu} C_{H,t}^{1-\nu} C_{F,t}^\nu$ denotes the consumption index, $C_{H,t} \equiv \left[\int_0^1 C_{H,t}(j)^{\frac{\epsilon-1}{\epsilon}} dj \right]^{\frac{\epsilon}{\epsilon-1}}$ is the index of domestic goods consumption, $C_{F,t} \equiv \left[\int_0^1 C_{F,t}(j)^{\frac{\epsilon-1}{\epsilon}} dj \right]^{\frac{\epsilon}{\epsilon-1}}$ is the quantity of a composite foreign good consumed, $\epsilon > 0$ is the elasticity of substitution between goods, $\nu \in [0, 1]$ is the measure of openness, $N_t \equiv \int_0^1 N_t(j) dj$ is the hours of labor, Z_t is the exogenous preference shifter, $\beta \equiv \frac{1}{1+\rho} \in (0, 1)$ denotes the subjective discount factor, and ρ is the rate of time preference with $P_{H,t} \equiv \left[\int_0^1 P_{H,t}(j)^{1-\epsilon} dj \right]^{\frac{1}{1-\epsilon}}$ and $P_{F,t} \equiv \left[\int_0^1 P_{F,t}(j)^{1-\epsilon} dj \right]^{\frac{1}{1-\epsilon}}$.

Period utility is

$$\mathcal{U}(C_t, L_t, N_t; Z_t) \equiv [U(C_t, L_t) - V(N_t)] Z_t,$$

with $V(\cdot)$ increasing and convex and $U(\cdot)$ increasing and concave.

Optimal allocation of any given expenditure within each category of goods yields the following demand function:

$$C_{H,t}(j) = \left(\frac{P_{H,t}(j)}{P_{H,t}} \right)^{-\epsilon} C_{H,t} \quad ; \quad C_{F,t}(j) = \left(\frac{P_{F,t}(j)}{P_{F,t}} \right)^{-\epsilon} C_{F,t}, \quad (\text{A.2})$$

for all j . The optimal allocation of expenditures between domestic and foreign goods implies that

$$C_{H,t} = (1 - \nu) \mathcal{S}_t^\nu C_t \quad ; \quad C_{F,t} = \nu \mathcal{S}_t^{-(1-\nu)} C_t. \quad (\text{A.3})$$

The sequence of budget constraints is as follows:

$$\begin{aligned} P_t C_t + B_{H,t} + \mathcal{E}_t B_{H,t}^* + M_t &= B_{H,t-1} (1 + i_{t-1}) + \mathcal{E}_t B_{H,t-1}^* (1 + i_{t-1}^*) + M_{t-1} + W_t N_t \\ &\quad - P_t T R_t + D_t, \end{aligned}$$

where $B_{H,t}$ denotes the nominal riskless one-period domestic government bond in domestic currency units held by domestic households, $B_{H,t}^*$ is the nominal riskless one-period foreign government bond in units of foreign currency held by domestic households, i_t is the domestic nominal interest rate, i_t^* is the foreign nominal interest rate, \mathcal{E}_t is the nominal exchange rate (the price of foreign currency in units of the domestic currency), W_t is the nominal wage, and D_t is the nominal dividend. Note that $B_t = B_{H,t} + B_{F,t}$, where $B_{F,t}$ denotes nominal riskless one-period domestic government bonds in units of the domestic currency held by foreign households.

Dividing both sides of the previous expression by the CPI P_t yields:

$$\begin{aligned} C_t + \mathcal{B}_{H,t} + \mathcal{Q}_t \mathcal{B}_{H,t}^* + L_t &= \Pi_t^{-1} \mathcal{B}_{H,t-1} (1 + i_{t-1}) + (\Pi_t^*)^{-1} \mathcal{Q}_t \mathcal{B}_{H,t-1}^* (1 + i_{t-1}^*) \\ &\quad + \Pi_t^{-1} L_{t-1} + \frac{W_t}{P_t} N_t - T R_t + \frac{D_t}{P_t}, \end{aligned} \quad (\text{A.4})$$

where $\mathcal{B}_{H,t} \equiv \frac{B_{H,t}}{P_t}$ denotes real domestic government debt held by domestic households, $\mathcal{B}_t^* \equiv \frac{B_{H,t}^*}{P_t^*}$ is real foreign government debt held by domestic households, $\mathcal{Q}_t \equiv \frac{\mathcal{E}_t P_t^*}{P_t}$ is the real exchange rate (the ratio of the CPI expressed in domestic currency), $\Pi_t^* \equiv \frac{P_t^*}{P_{t-1}^*}$ denotes the (gross) foreign inflation, P_t^* is the foreign price index, and $\Pi_t \equiv \frac{P_t}{P_{t-1}}$ is (gross) CPI inflation.

Assuming complete international financial markets, the equilibrium price (in domestic currency units) of a riskless bond denominated in a foreign currency is $(1 + i_t^*)^{-1} = \mathcal{Q}_{t,t+1} \mathcal{E}_{t+1}$, where $\mathcal{Q}_{t,t+1}$ denotes the price of a one-period discount bond paying off one domestic currency unit. We combine

the previous pricing equation with the domestic bond-pricing equation, $(1 + i_t)^{-1} = Q_{t,t+1}$, to obtain a version of the uncovered interest parity (UIP) condition:

$$Q_{t,t+1} \left[(1 + i_t) - (1 + i_t^*) \left(\frac{\mathcal{E}_{t+1}}{\mathcal{E}_t} \right) \right] = 0. \quad (\text{A.5})$$

We define $\mathcal{A}_t \equiv \left[(1 + i_{t-1}) \mathcal{B}_{H,t-1} + Q_{t-1} \mathcal{B}_{H,t-1}^* \frac{\mathcal{E}_t}{\mathcal{E}_{t-1}} (1 + i_{t-1}^*) + L_{t-1} \right] \Pi_t^{-1}$ as the representative household's real financial wealth at the beginning of period t . Therefore, we can rewrite Eq. (A.4) as

$$C_t + \frac{1}{1 + i_t} \mathcal{A}_{t+1} \Pi_{t+1} + L_t \left(1 - \frac{1}{1 + i_t} \right) = \mathcal{A}_t + \frac{W_t}{P_t} N_t - TR_t + \frac{D_t}{P_t}, \quad (\text{A.6})$$

where we assume the standard solvency constraint $\lim_{k \rightarrow \infty} \Lambda_{t,t+k} \mathcal{A}_{t+k} \geq 0$, ruling out the Ponzi scheme.

Households maximize Eq. (A.1) subject to Eq. (A.6) and have the following optimality conditions:

$$U_{c,t} = \beta (1 + i_t) \Pi_{t+1}^{-1} U_{c,t+1} \frac{Z_{t+1}}{Z_t}, \quad (\text{A.7})$$

$$\frac{W_t}{P_t} = \frac{V_{n,t}}{U_{c,t}}, \quad (\text{A.8})$$

$$h \left(\frac{L_t}{C_t} \right) = \frac{i_t}{1 + i_t}, \quad (\text{A.9})$$

with $h \left(\frac{L_t}{C_t} \right) \equiv \frac{U_{l,t}}{U_{c,t}}$. $h \left(\frac{L}{C} \right) \equiv \frac{U_l}{U_c}$ is a continuously decreasing function that satisfies $h(\bar{\chi}) = 0$ for some $0 < \bar{\chi} < \infty$. This guarantees that the demand for the real money balance is bounded as the interest rate approaches zero, with a satiation point attained at $L = \bar{\chi}C$. Eqs. (A.7), (A.8), and (A.9) are the consumption Euler equation and the intertemporal optimality condition that determine the labor supply under the assumption of a competitive labor market and money demand schedule, respectively. These optimality conditions require the TVC $\lim_{k \rightarrow \infty} \Lambda_{t,t+k} \mathcal{A}_{t+k} = 0$.

A.2 International Risk-sharing Condition

Assuming a complete financial market, a condition analogous to Eq. (A.4) must hold for a representative household in a foreign country. By combining this condition with Eq. (A.7), UIP, and

the definition of the real exchange rate, we obtain the international risk-sharing condition

$$U_{c,t}^{-1} = \vartheta (U_{c,t}^*)^{-1} \mathcal{Q}_t \frac{Z_t}{Z_t^*}, \quad (\text{A.10})$$

where $U_{c,t}^*$ denotes the counterpart of $U_{c,t}$ in the foreign country, Z_t^* denotes the foreign exogenous preference shifter, and ϑ is a constant that depends on initial conditions.

We assume LOOP; that is, $P_{F,t}(j) = \mathcal{E}_t P_{F,t}^*(j)$ for all j , where $P_{F,t}^*(j)$ denotes the price of foreign good j in units of foreign currency. By integrating all the goods, we have

$$P_{F,t} = \mathcal{E}_t P_{F,t}^*, \quad (\text{A.11})$$

where $P_{F,t}^*$ denotes the foreign currency price of the foreign goods. Our treatment of the rest of the world as an (approximately) closed economy (with goods produced in a small open economy representing a negligible fraction of the world's consumption basket) implies that the foreign price index coincides with the foreign currency prices of foreign goods; that is, $P_t^* = P_{F,t}^*$.

Substituting the definition of CPI for the real exchange rate yields

$$\mathcal{Q}_t = S_t^{1-\nu}. \quad (\text{A.12})$$

This implies that the assumption of complete markets at the international level leads to a simple relationship between consumption at home and abroad, and TOT. Substituting Eq. (A.12) into Eq. (A.10) gives

$$U_{c,t}^{-1} = \vartheta (U_{c,t}^*)^{-1} S_t^{1-\nu} \frac{Z_t}{Z_t^*}. \quad (\text{A.13})$$

A.3 Domestic Producers

A typical domestic firm produces a differentiated good using the following technology:

$$Y_t(j) = N_t(j)^{1-\alpha},$$

where $Y_t(j)$ is the output of generic good j and α denotes the index of decreasing returns to labor. The index for aggregate domestic output is $Y_t \equiv \left[\int_0^1 Y_t(j)^{\frac{\epsilon-1}{\epsilon}} dj \right]^{\frac{\epsilon}{\epsilon-1}}$. By integrating the previous

expression, we obtain

$$N_t^{1-\alpha} = Y_t \left[\int_0^1 \left(\frac{P_{H,t}(j)}{P_{H,t}} \right)^{-\frac{\epsilon}{1-\alpha}} dj \right]^{1-\alpha}, \quad (\text{A.14})$$

where $\int_0^1 \left(\frac{P_{H,t}(j)}{P_{H,t}} \right)^{-\frac{\epsilon}{1-\alpha}} dj$ denotes price dispersion.

In each period, a subset of firms of measure $1-\theta$, with $\theta \in [0, 1]$ being an index of price rigidities drawn randomly from the population, reoptimizes the price of their goods, subject to a sequence of isoelastic demand schedules for the latter. The remaining θ firms retain their prices. In other words, the firms are subject to Calvo pricing. Prices are set in the domestic currency, domestic and export markets share the same price, and the LOOP also applies to exports.

The first-order necessary condition (FONC) for domestic producers is

$$\sum_{k=0}^{\infty} \theta^k \left[\Lambda_{t,t+k} \left(\frac{1}{P_{t+k}} \right) Y_{t+k|t} \left(\tilde{P}_{H,t} - \mathcal{M} MC_{t+k|t}^n \right) \right] = 0, \quad (\text{A.15})$$

where $\mathcal{M} \equiv \frac{\epsilon}{\epsilon-1}$ denotes the (desired) price markup, $Y_{t+k|t} \equiv Y_t \left(\frac{\tilde{P}_{H,t}}{P_{H,t+k}} \right)^{-\epsilon}$ is output in period $t+k$ for a firm that last reset its price in period t , $\tilde{P}_{H,t}$ is the price set in period t by firms reoptimizing their price in that period, $MC_{t+k|t}^n$ is the nominal marginal cost in period $t+k$ for a firm that last reset its price in period t , and $MC_t^n \equiv W_t \left(\frac{N_t^\alpha}{1-\alpha} \right)$ is the nominal marginal cost.

A.4 Demand for Exports and Global Shocks

The demand for exports of domestic goods j is

$$EX_t(j) = \left(\frac{P_{H,t}(j)}{P_{H,t}} \right)^{-\epsilon} EX_t, \quad (\text{A.16})$$

where EX_t is the aggregate export index.

Following Galí and Monacelli (2016), aggregate exports are

$$EX_t = \nu S_t Y_t^*, \quad (\text{A.17})$$

where Y_t^* denotes (per-capita) world output.

A.5 The Market-clearing Condition

The market-clearing condition is

$$Y_t(j) = C_{H,t}(j) + EX_t(j) + G_t(j).$$

Plugging Eqs. (A.2), (A.3), (A.16), (A.17), and (A.10) into the previous expression, we obtain

$$Y_t = (1 - \nu) S_t^\nu C_t + \nu S_t Y_t^* + G_t, \quad (\text{A.18})$$

where we use the optimal allocation of the output $Y_t(j) = \left(\frac{P_{H,t}(j)}{P_{H,t}}\right)^{-\epsilon} Y_t$. We assume that $Y_t^* = C_t^*$, where C_t^* denotes the (per capita) world consumption.

Appendix B: Log-linearized Equilibrium Conditions

This appendix presents the full set of log-linearized equilibrium conditions. The notation follows Table C1.

B.1 Core Equilibrium Conditions

International Risk-sharing Condition

$$\hat{\xi}_t = -(1 - \nu) s_t + \hat{\xi}_t^* - \zeta_t. \quad (\text{B.1})$$

Market-clearing Condition

$$\hat{y}_t = \nu(2 - \nu) s_t + (1 - \nu) \hat{c}_t + \nu \hat{y}_t^* + \hat{g}_t. \quad (\text{B.2})$$

Consumption Euler Equation

$$\hat{\xi}_t = \hat{\xi}_{t+1} + \hat{i}_t - \pi_{t+1} - \hat{\rho}_t. \quad (\text{B.3})$$

Marginal Utility of Consumption

$$\hat{\xi}_t = -\sigma \hat{c}_t + v \hat{l}_t. \quad (\text{B.4})$$

New Keynesian Phillips Curve (NKPC)

$$\pi_{H,t} = \beta\pi_{H,t+1} - \kappa\hat{\mu}_t, \quad (\text{B.5})$$

where $\kappa \equiv \frac{(1-\theta\beta)(1-\theta)\Theta}{\theta}$ and $\Theta \equiv \frac{1-\alpha}{(1-\alpha)+\alpha\epsilon}$.

Price Markup Gap

$$\hat{\mu}_t = \hat{\xi}_t - \frac{\alpha + \varphi}{1 - \alpha} \hat{y}_t - \nu s_t. \quad (\text{B.6})$$

B.2 Money and Fiscal Variables

Money Demand Schedule

$$\hat{l}_t = \hat{c}_t - \eta \hat{i}_t. \quad (\text{B.7})$$

Logarithmic First Differential of the Definition of the Real Money Balance

$$\hat{l}_{t-1} = \hat{l}_t + \pi_t - \Delta m_t. \quad (\text{B.8})$$

Consolidated Government Budget Constraint

$$\hat{b}_t = (1 + \rho) \hat{b}_{t-1} + (1 + \rho) b \hat{i}_{t-1} - (1 + \rho) b \pi_t + \hat{g}_t - \hat{tr}_t - \chi \Delta m_t. \quad (\text{B.9})$$

B.3 Price and Exchange Rate Definitions

Combination of the Logarithmic First Differential of the Definition of the CPI and TOT

$$\pi_t = \pi_{H,t} + \nu (s_t - s_{t-1}). \quad (\text{B.10})$$

Definition of the Trade Balance

$$\widehat{nx}_t = \hat{y}_t - \nu s_t - \hat{c}_t - \hat{g}_t. \quad (\text{B.11})$$

Definition of the TOT

$$s_t = e_t + p_t^* - p_{H,t}. \quad (\text{B.12})$$

Definition of Domestic Inflation

$$\pi_{H,t} = p_{H,t} - p_{H,t-1}. \quad (\text{B.13})$$

Definition of Import Inflation

$$\pi_{F,t} = p_{F,t} - p_{F,t-1}. \quad (\text{B.14})$$

Combination of the (Logarithmic) Definition of the TOT with the (Logarithmic) Definition of Domestic and Import Inflation

$$\pi_{F,t} = s_t - s_{t-1} + \pi_{H,t}. \quad (\text{B.15})$$

B.4 Parameter Definitions

The parameters are defined as follows:

- $\beta \equiv \frac{1}{1+\rho} \in (0, 1)$: subjective discount factor
- ρ : rate of time preference
- $\mu \equiv \log \mathcal{M}$: constant (desired) price markup
- $\eta \equiv \frac{\epsilon_{lc}}{\rho}$, where $\epsilon_{lc} \equiv \frac{1}{\sigma_l + v}$: semi-elasticity of money demand
- $\sigma_l \equiv \frac{U_{ll}L}{U_l}$: elasticity of substitution between consumption and real balances
- $\chi \equiv \frac{L}{Y}$: steady-state inverse velocity of money
- $b \equiv \frac{B}{Y}$: steady-state share of government debt to output
- $\varphi \equiv \frac{V_{nn}N}{V_n}$: curvature of labor disutility
- $v \equiv \frac{U_{cl}L}{U_c}$: separability of real balances
- $\sigma \equiv -\frac{U_{cc}C}{U_c}$: relative risk aversion

We assume $Z_t^* = 1$ and $Z_{t+1} = Z_t^\varrho$ with $\varrho = 0$. Therefore, $\hat{\rho}_t = \log Z_t = \zeta_t$ where $\hat{\rho}_t$ and ζ_t denote the demand and risk-sharing shocks, respectively.

Appendix C: Variable Definitions

Table C1 presents the definitions of all the logarithmic variables used in the analysis.

Table C1: Definition of Logarithmic Variables

Variable	Definition	Variable	Definition
\hat{y}_t	$\log\left(\frac{Y_t}{\bar{Y}}\right)$	\hat{b}_t	$\frac{\bar{B}_t - \bar{B}}{\bar{Y}}$
\hat{c}_t	$\log\left(\frac{C_t}{\bar{C}}\right)$	\hat{tr}_t	$\frac{TR_t - TR}{\bar{Y}}$
$\hat{\rho}_t$	$-\log\left(\frac{Z_{t+1}}{Z_t}\right)$	\widehat{nx}_t	$\log\left[\left(\frac{NX_t}{P_{H,t}}\right)/Y\right]$
$p_{H,t}$	$\log P_{H,t}$	ζ_t	$-\log\left(\frac{Z_t^*}{Z_t}\right)$
$p_{F,t}$	$\log P_{F,t}$	\hat{y}_t^*	$\log\left(\frac{Y_t^*}{\bar{Y}^*}\right)$
s_t	$\log \mathcal{S}_t$	ξ_t^*	$\log\left(\frac{U_{c,t}^*}{\bar{U}_c^*}\right)$
\hat{g}_t	$\frac{G_t}{\bar{Y}}$	p_t^*	$\log P_t^*$
ξ_t	$\log\left(\frac{U_{c,t}}{\bar{U}_c}\right)$	e_t	$\log \mathcal{E}_t$
π_t	$\log \Pi_t$	$\pi_{H,t}$	$\log \Pi_{H,t}$
\hat{i}_t	$\log\left(\frac{1+i_t}{1+\rho}\right)$	$\pi_{F,t}$	$\log \Pi_{F,t}$
m_t	$\log M_t$	μ_t	$-\log MC_t$
\hat{l}_t	$\log\left(\frac{L_t}{\bar{L}}\right)$	$\hat{\mu}_t$	$\mu_t - \mu$
\widehat{sp}_t	$\log\left(\frac{SP_t}{\bar{SP}}\right)$		

Note: $MC_t \equiv \frac{MC_t^n}{P_{H,t}}$ denotes the real marginal cost. $\hat{\mu}_t$ is the markup gap.

Appendix D: Derivation of the Log-linearized FTPL Equilibrium Restriction

This appendix demonstrates the formal equivalence between the intertemporal valuation relation Eq. (4) in the main text and the log-linearized FTPL equilibrium restriction Eq. (D.3). The equivalence holds under the transversality condition $\lim_{j \rightarrow \infty} \beta^{t+j+1} R_{t+j} (B_{t+j} + L_{t+j}) = 0$ imposed in Section 3.1.1 and the households' Euler equation derived in Online Appendix A. The derivation proceeds in four steps.

Step D.1: From the intertemporal valuation relation to the second-order difference equation. Starting from Eq. (4) in the main text,

$$U_{c,t}Z_tR_{t-1}(B_{t-1} + L_{t-1}) = \left\{ \sum_{h=0}^{\infty} \beta^h U_{c,t+h}Z_{t+h}SP_{t+h} + \sum_{h=0}^{\infty} \beta^{h-1} U_{c,t+h-1}Z_{t+h-1} \left(\frac{i_{t+h-1}}{1 + i_{t+h-1}} \right) L_{t+h-1} \right\} \Pi_t,$$

we take the period- t equation and subtract β times the period- $(t + 1)$ equation (shifted forward), exploiting the telescoping structure of the present-value sums. The households' Euler equation $U_{c,t}Z_t = \beta R_t U_{c,t+1}Z_{t+1}\Pi_{t+1}$ allows the remaining terms to be rearranged, yielding

$$\begin{aligned} U_{c,t}Z_tSP_t + \beta^{-1}U_{c,t-1}Z_{t-1}\frac{i_{t-1}}{1 + i_{t-1}}L_{t-1} \\ = U_{c,t}Z_t(1 + i_{t-1})(B_{t-1} + L_{t-1})\Pi_t^{-1} - \beta U_{c,t+1}Z_{t+1}(1 + i_t)(B_t + L_t)\Pi_{t+1}^{-1}. \end{aligned} \quad (\text{D.1})$$

Eq. (D.1) is the second-order difference equation referenced in Section 3.1.2. It is algebraically equivalent to Eq. (4) in the main text under the transversality condition.

Step D.2: Interpretation of Eq. (D.1). The left-hand side of Eq. (D.1) represents the consolidated government's revenue in period t , in terms of the marginal utility of consumption: the first term is the fiscal surplus, and the second term is the interest revenue derived from supplying liquidity services in period $t - 1$. The right-hand side represents the burden of redeeming consolidated government debt (including real money balances) with interest payments in period t , net of the value of newly issued debt (again including real money balances) carried into period $t + 1$.

Step D.3: Log-linearization around the zero-inflation steady state. Log-linearizing Eq. (D.1) around the steady state characterized in Section 4, and using the log-linearized Euler equation to eliminate the marginal utility of consumption, we obtain

$$\hat{i}_{t-1} + \frac{b(1-\beta)}{\chi\beta}\hat{s}p_t = \frac{b+\chi}{\chi}\hat{i}_{t-1} + \frac{1}{\chi}\hat{b}_{t-1} + \frac{b(1-\beta)^2 + \chi\beta^2}{\chi\beta}\hat{l}_{t-1} - \frac{\beta}{\chi}\hat{b}_t - \beta\hat{l}_t - \frac{b+\chi\beta}{\chi}\pi_t. \quad (\text{D.2})$$

Step D.4: Substituting the fiscal surplus definition. Substituting the logarithmic definition of the fiscal surplus,

$$\hat{s}p_t = \frac{\beta}{b(1-\beta)}\hat{t}r_t - \frac{\beta}{1-\beta}\hat{g}_t,$$

into the left-hand side of Eq. (D.2) gives

$$\hat{tr}_t = \hat{b}_{t-1} + \hat{b}_{t-1} + \frac{b(1-\beta)^2 + \chi\beta^2}{\beta} \hat{l}_{t-1} - \beta \hat{b}_t - \beta \chi \hat{l}_t - (b + \chi\beta)\pi_t + \hat{g}_t. \quad (\text{D.3})$$

This completes the demonstration that Eq. (D.3) (which corresponds to Eq. (8) in the main text) is the log-linear equivalent of the intertemporal valuation relation Eq. (4) in the main text, derived via the second-order difference equation Eq. (D.1).

Appendix E: Second-order Welfare Approximation (Detailed Derivation)

This appendix provides a detailed derivation of the welfare cost function used in Section 6.5 of the main text. We take a second-order Taylor approximation of household utility around the zero-inflation steady state and express the resulting loss function in terms of the welfare-relevant output gap and domestic inflation. The derivation adapts the framework of Okano and Eguchi (2024), Punzo and Rossi (2023), and Galí (2020) to a small open economy with log-utility preferences.

Step E.1: Second-order approximation of household utility. Following the welfare analysis in the New Keynesian literature, we evaluate welfare using the consumption-labor component of household utility and omit the nonseparability of real balances ($v = 0$) in the welfare approximation. This isolates the welfare costs associated with the welfare-relevant output gap and domestic price dispersion.

Under log utility (with relative risk aversion $\sigma = 1$), the consumption-labor component of household utility is

$$\tilde{U}_t = \ln C_t - \frac{1}{1+\varphi} N_t^{1+\varphi}, \quad (\text{E.1})$$

where φ is the curvature of labor disutility. After eliminating the first-order terms via the undetermined-coefficients method described in Steps E.2–E.3 below, the policy-relevant component of the second-order approximation can be written as

$$\sum_{t=0}^{\infty} \beta^t \left(\frac{\tilde{U}_t - \tilde{U}}{\tilde{U}_{CC}} \right) = \sum_{t=0}^{\infty} \beta^t \left\{ \tilde{\Phi} \hat{y}_t - \frac{\nu(2-\nu)}{1-\nu} s_t \right\} - \sum_{t=0}^{\infty} \beta^t \left(\frac{\Omega_y}{2} \tilde{x}_t^2 + \frac{\Omega_\pi}{2} \pi_{H,t}^2 \right) + \text{s.o.t.i.p.} + \text{t.i.p.} + \Upsilon_0 + o(\|\xi\|^3), \quad (\text{E.2})$$

where $\tilde{x}_t \equiv \hat{y}_t - \hat{y}_t^e$ is the welfare-relevant output gap and $\tilde{\Phi}$ is a parameter combining structural features of the economy. The efficient output level \hat{y}_t^e is defined as the allocation that would prevail under flexible prices and the efficient (zero-markup) steady state, holding exogenous shocks fixed. Thus, the welfare-relevant gap measures deviations of output from the efficient allocation.

Step E.2: Second-order approximated structural equations. To make explicit how the first-order terms are eliminated, we record the second-order approximations of the structural equations used in the undetermined-coefficients argument in Step E.3. The expressions below retain the quadratic and cross-product terms that enter the policy-relevant component of welfare through the Θ_i substitution.

The second-order approximated aggregate supply (Phillips curve) equation is

$$\begin{aligned} \bar{\nu} = \kappa \sum_{k=0}^{\infty} (\beta\theta)^k & \left[\hat{w}_{t+k}^r + \alpha \hat{n}_{t+k} + (\hat{w}_{t+k}^r)^2 + \alpha^2 \hat{n}_{t+k}^2 + 2\alpha \hat{w}_{t+k}^r \hat{n}_{t+k} \right. \\ & - \hat{c}_{t+k} \hat{w}_{t+k}^r - \alpha \hat{c}_{t+k} \hat{n}_{t+k} + \hat{w}_{t+k}^r z_{t+k} + \alpha \hat{n}_{t+k} z_{t+k} \\ & \left. + \hat{y}_{t+k} \hat{w}_{t+k}^r + \alpha \hat{y}_{t+k} \hat{n}_{t+k} - s_{t+k} \hat{w}_{t+k}^r - \nu \alpha s_{t+k} \hat{n}_{t+k} \right] \\ & + \frac{\kappa \varepsilon (2 - \alpha)}{2} \sum_{k=0}^{\infty} (\beta\theta)^k \pi_{H,t+k}^2 + o(\|\xi\|^3), \quad (\text{E.3}) \end{aligned}$$

where $\bar{\nu}$ is a constant related to steady-state price dispersion, \hat{w}_t^r denotes the log-deviation of the real wage, \hat{n}_t is log employment, and z_t is the exogenous preference shock.

The second-order approximated wage equation is

$$\begin{aligned} 0 = \sum_{t=0}^{\infty} \beta^t \mathbb{E}_0 & \left\{ -\hat{w}_t^r + \nu s_t + \hat{c}_t + \varphi \hat{n}_t + \frac{\nu^2}{2} s_t^2 + \frac{1}{2} \hat{c}_t^2 + \frac{\varphi^2}{2} \hat{n}_t^2 \right. \\ & \left. + \nu s_t \hat{c}_t + \nu \varphi s_t \hat{n}_t + \varphi \hat{c}_t \hat{n}_t \right\} + \text{t.i.p.} + o(\|\xi\|^3), \quad (\text{E.4}) \end{aligned}$$

which equates the marginal rate of substitution between labor and consumption to the real wage.

The second-order approximated government solvency condition is

$$\begin{aligned} \bar{\omega} = (1 - \beta) \sum_{t=0}^{\infty} \beta^t \mathbb{E}_0 & \left\{ -\hat{c}_t + \frac{\beta}{b(1 - \beta)} \hat{t}r_t + \frac{1}{2} \left[\frac{\beta}{b(1 - \beta)} \right]^2 (\hat{t}r_t)^2 + \frac{1}{2} \hat{c}_t^2 \right. \\ & \left. - \frac{\beta}{b(1 - \beta)} \hat{c}_t \hat{t}r_t - \hat{c}_t z_t + \frac{\beta}{b(1 - \beta)} z_t \hat{t}r_t \right\} + \text{t.i.p.} + o(\|\xi\|^3), \quad (\text{E.5}) \end{aligned}$$

where $\bar{\omega}$ is a constant and the government's budget constraint ensures fiscal sustainability.

The second-order approximated market-clearing condition is

$$0 = \sum_{t=0}^{\infty} \beta^t \mathbb{E}_0 \left\{ -\hat{y}_t + (1-\nu)\hat{c}_t + \nu(2-\nu)s_t + \frac{1-\nu}{2}\hat{c}_t^2 + \frac{\nu[1+\nu(1-\nu)]}{2}s_t^2 + (1-\nu)\hat{c}_t s_t \right\} + \text{t.i.p.} + o(\|\xi\|^3), \quad (\text{E.6})$$

which states that aggregate demand equals aggregate supply (adjusted for openness).

The second-order approximated international risk-sharing condition is

$$0 = \sum_{t=0}^{\infty} \beta^t \mathbb{E}_0 [\hat{c}_t - (1-\alpha)s_t] + \text{t.i.p.} + o(\|\xi\|^3), \quad (\text{E.7})$$

which pins down the relationship between domestic consumption and the terms of trade given international asset markets.

The second-order approximated production function is

$$0 = \sum_{t=0}^{\infty} \beta^t \left(-\hat{n}_t + \frac{1}{1-\alpha}\hat{y}_t + \frac{1}{2(1-\alpha)}\hat{y}_t^2 + \frac{\varepsilon}{2\kappa}\pi_{H,t}^2 \right), \quad (\text{E.8})$$

which relates labor input to output given the production technology.

Step E.3: Elimination of first-order terms via undetermined coefficients. The first-order components of equations (E.3)–(E.8) form a linear system that determines the first-order dynamics. The key insight is that by choosing undetermined coefficient values, we can eliminate all first-order terms in output, consumption, labor, terms of trade, and fiscal variables.

The first-order system for the undetermined coefficients $\Theta_1, \Theta_2, \Theta_3, \Theta_4, \Theta_5, \Theta_6$ is given by:

$$\begin{bmatrix} \tilde{\Phi} \\ 0 \\ -\frac{\nu(2-\nu)}{1-\nu} \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} -\Theta_3 + \frac{1}{1-\alpha}\Theta_6 \\ \frac{\beta}{b(1-\beta)}\Theta_1 \\ \nu(2-\nu)\Theta_3 - (1-\alpha)\Theta_4 + \nu\Theta_5 \\ -\Theta_1 + (1-\nu)\Theta_3 + \Theta_4 + \Theta_5 \\ \Theta_2 - \Theta_5 \\ \alpha\Theta_2 + \varphi\Theta_5 - \Theta_6 \end{bmatrix}, \quad (\text{E.9})$$

where the definitions of structural parameters (Γ_1, Γ_2 , etc.) determine the coefficients on the right-hand side.

The solutions to this system are:

$$\Theta_1 = 0, \quad (\text{E.10})$$

$$\Theta_2 = \frac{(1-\alpha)\Gamma_1}{(1-\nu)\Gamma_2}, \quad (\text{E.11})$$

$$\Theta_3 = \frac{(\alpha+\varphi)\Gamma_1 - (1-\nu)\Gamma_2\tilde{\Phi}}{(1-\nu)\Gamma_2}, \quad (\text{E.12})$$

$$\Theta_4 = -\frac{(1-\nu)\Gamma_3 - (1-\alpha)\Gamma_1}{(1-\nu)\Gamma_2}, \quad (\text{E.13})$$

$$\Theta_5 = \frac{(1-\alpha)\Gamma_1}{(1-\nu)\Gamma_2}, \quad (\text{E.14})$$

$$\Theta_6 = \frac{(1-\alpha)(\alpha+\varphi)\Gamma_1}{(1-\nu)\Gamma_2}. \quad (\text{E.15})$$

By choosing these coefficients, all first-order terms in the policy-relevant variables cancel out, up to terms independent of policy.

Step E.4: Derivation of the quadratic welfare expression. Substituting the undetermined coefficients back into the system, the first-order terms vanish and we are left with a purely quadratic expression in deviations. The quadratic welfare expression can be written in the intermediate form:

$$\sum_{t=0}^{\infty} \beta^t \left\{ \frac{\Gamma_{11}}{2[\Gamma_3(1-\alpha)]^2} \left[\hat{y}_t^2 - 2\hat{y}_t \left(\frac{\Gamma_{13}}{\Gamma_{12}} \hat{g}_t + \frac{\Gamma_{14}}{\Gamma_{12}} z_t \right) \right] + \frac{\varepsilon\Gamma_{15}}{2\kappa} \pi_{H,t}^2 \right\}, \quad (\text{E.16})$$

where \hat{g}_t and z_t represent exogenous fiscal and preference shocks, respectively. Completing the square to express welfare in terms of the gap from the efficient allocation yields:

$$\sum_{t=0}^{\infty} \beta^t \left(\frac{\tilde{U}_t - \tilde{U}}{\tilde{U}_C C} \right) = - \sum_{t=0}^{\infty} \beta^t \left(\frac{\Omega_y}{2} \tilde{x}_t^2 + \frac{\Omega_\pi}{2} \pi_{H,t}^2 \right) + \text{s.o.t.i.p.} + \text{t.i.p.} + \Upsilon_0 + o(\|\xi\|^3), \quad (\text{E.17})$$

where the welfare-relevant output gap is $\tilde{x}_t = \hat{y}_t - \hat{y}_t^e$.

Step E.5: Welfare cost function. Following the standard long-run average-loss representation in the literature, we take the limit $\beta \rightarrow 1$ in Eq. (E.17) and focus on the policy-relevant quadratic terms to obtain the welfare cost function:

$$\mathcal{L} = \frac{\Omega_y}{2} \text{var}(\tilde{x}_t) + \frac{\Omega_\pi}{2} \text{var}(\pi_{H,t}), \quad (\text{E.18})$$

which is the welfare criterion used in Section 6.5 of the main text. In the quantitative calculations, however, the transition paths are generated under the calibrated value $\beta = 0.995$.

In the welfare comparison, the discounted second moment is computed as:

$$\text{var}(z_t) \equiv (1 - \beta) \sum_{t=0}^{T_{\text{sim}}} \beta^t z_t^2, \quad (\text{E.19})$$

where T_{sim} is the terminal period of the simulation. This measure corresponds to the discounted second moment along the simulated transition path.

Welfare improvement for regime r is measured relative to the *no response* with CIT baseline:

$$WI^r = 100 \times \frac{\mathcal{L}^{\text{baseline}} - \mathcal{L}^r}{\mathcal{L}^{\text{baseline}}}, \quad (\text{E.20})$$

where $\mathcal{L}^{\text{baseline}}$ denotes the welfare cost under *no response* with CIT and \mathcal{L}^r is the welfare cost under regime r . A positive value indicates that regime r improves welfare relative to the baseline.

Step E.6: Parameter definitions and structural coefficients. The welfare cost coefficients depend on the structural parameters of the model. Under log utility ($\sigma = 1$), we first define the steady-state markup and distortion parameters:

$$\mathcal{M} = \frac{\varepsilon}{\varepsilon - 1}, \quad (\text{E.21})$$

$$\Phi = 1 - \frac{1 - \alpha}{\mathcal{M}}, \quad (\text{E.22})$$

$$\tilde{\Phi} = \frac{1}{1 - \nu} - \frac{1 - \Phi}{1 - \alpha}, \quad (\text{E.23})$$

$$\Theta = \frac{1 - \alpha}{(1 - \alpha) + \alpha\varepsilon}. \quad (\text{E.24})$$

The parameter Φ measures the steady-state price markup distortion, while $\tilde{\Phi}$ reflects how this distortion translates into welfare consequences for a small open economy.

The slope of the New Keynesian Phillips curve is given by:

$$\kappa = \frac{(1 - \theta\beta)(1 - \theta)}{\theta} \Theta, \quad (\text{E.25})$$

which captures the degree of price stickiness (θ) and price elasticity of demand.

The building-block, higher-order, and composite Γ coefficients that enter the welfare cost expression, together with the intermediate Θ coefficients defined in Eqs. (E.10)–(E.15), are summarized

in Table E1.

Table E1: Definitions of the Blocks of Parameters

Parameters	Definitions
Γ_1	$(1 - \nu)[\nu(2 - \nu) + (1 - \nu)(1 - \alpha)]\tilde{\Phi} - \nu(2 - \nu)$
Γ_2	$\nu(2 - \nu)(\alpha + \varphi) + (1 - \alpha)[(1 - \nu)(\alpha + \varphi) + (1 - \alpha)] + (1 - \alpha)\nu$
Γ_3	$\nu(2 - \nu) + (1 - \nu)^2$
Γ_4	$(1 - \alpha)(2\nu - 1) + \Gamma_3[(1 - \nu)^2 + 2]$
Γ_5	$\Gamma_3[\varphi(1 + \nu) - \alpha + \Gamma_3(\varphi + \alpha)] + 2\alpha$
Γ_6	$3\varphi^2\Gamma_3 + 2\alpha^2\Gamma_3 + 4\alpha\varphi$
Γ_7	$(1 - \nu)^2[2 + (1 - \nu)\Gamma_3] + \nu[1 + \nu(1 - \nu)]$
Γ_8	$(1 - \alpha) - \Gamma_3[\varphi(3 - \nu) + 1 - \alpha]$
Γ_9	$\nu[1 + \nu(1 - \nu)] + (1 - \nu)^2[(1 - \nu) + 2]$
Γ_{10}	$-(1 - \alpha) - \Gamma_3(\varphi + \alpha)$
Γ_{11}	$\Theta_2[(1 - \alpha)(\Gamma_4 + 2\Gamma_5) + \Gamma_3\Gamma_6] + \Theta_3(1 - \alpha)^2\Gamma_7 + \Theta_6\Gamma_3^2(1 - \alpha)$
Γ_{12}	$(1 - \Phi)(1 + \varphi)\Gamma_3^2 - \Gamma_{11}$
Γ_{13}	$-(1 - \alpha)[\Theta_2\Gamma_8 + \Theta_3(1 - \alpha)\Gamma_9]$
Γ_{14}	$-(1 - \alpha)\Theta_2\Gamma_3\Gamma_{10}$
Γ_{15}	$(1 - \Phi) - [\Theta_2\kappa^2(2 - \alpha) + \Theta_6]$
$\tilde{\Phi}$	$\frac{1}{1 - \nu} - \frac{(1 - \Phi)}{1 - \alpha}$

The welfare cost coefficients are then computed as:

$$\Omega_y = \frac{\Gamma_{12}}{(1 - \alpha)^2\Gamma_3^2}, \quad (\text{E.26})$$

$$\Omega_\pi = \frac{\varepsilon\Gamma_{15}}{2\kappa}. \quad (\text{E.27})$$

The coefficient Ω_y weights the welfare cost of the output gap, reflecting both the inefficiency from monopolistic competition and the open-economy structure. The coefficient Ω_π weights the welfare cost of inflation, which arises from the price-dispersion cost under Calvo pricing.

Step E.7: Calibration. Using the calibrated values from Table 2 of the main text — $\nu = 0.4$, $\beta = 0.995$, $\varphi = 5$, $\alpha = 0.25$, $\varepsilon = 9$, and $\theta = 0.75$ — we compute all the welfare-relevant parameters. *Markup and Distortion Parameters (Eqs. (E.21)–(E.24))*

$$\begin{aligned} \mathcal{M} &= \frac{9}{9 - 1} = 1.125, & \Phi &= 1 - \frac{0.75}{1.125} = 0.3333, \\ \tilde{\Phi} &= \frac{1}{0.6} - \frac{0.6667}{0.75} = 0.7778, & \Theta &= \frac{0.75}{0.75 + 0.25 \times 9} = 0.25. \end{aligned}$$

From Eq. (E.25): NKPC Slope

$$\kappa = \frac{(1 - 0.75 \times 0.995)(1 - 0.75)}{0.75} \times 0.25 = 0.0211.$$

Building-Block Coefficients (Table E1)

$$\Gamma_1 = 0.6[0.4 \times 1.6 + 0.6 \times 0.75] \times 0.7778 - 0.4 \times 1.6 = -0.1313,$$

$$\Gamma_2 = 0.4 \times 1.6 \times 5.25 + 0.75[0.6 \times 5.25 + 0.75] + 0.75 \times 0.4 = 6.5850,$$

$$\Gamma_3 = 0.4 \times 1.6 + 0.6^2 = 1.0000.$$

Undetermined Coefficients (Eqs. (E.10)–(E.15))

$$\Theta_2 = \frac{0.75 \times (-0.1313)}{0.6 \times 6.5850} = -0.0249, \quad \Theta_3 = \frac{5.25 \times (-0.1313) - 0.6 \times 6.5850 \times 0.7778}{0.6 \times 6.5850} = -0.9523,$$

$$\Theta_6 = \frac{0.75 \times 5.25 \times (-0.1313)}{0.6 \times 6.5850} = -0.1309.$$

Higher-Order Coefficients (Table E1)

$$\Gamma_4 = 0.75(0.8 - 1) + 1.0[0.36 + 2] = 2.2100,$$

$$\Gamma_5 = 1.0[5(1.4) - 0.25 + 1.0 \times 5.25] + 0.5 = 12.5000,$$

$$\Gamma_6 = 3 \times 25 \times 1.0 + 2 \times 0.0625 \times 1.0 + 4 \times 0.25 \times 5 = 80.1250,$$

$$\Gamma_7 = 0.36[2 + 0.6] + 0.4[1 + 0.24] = 1.4320, \quad \Gamma_8 = 0.75 - 1.0[5(2.4) + 0.75] = -13.0000,$$

$$\Gamma_9 = 0.4[1 + 0.24] + 0.36[0.6 + 2] = 1.4320, \quad \Gamma_{10} = -0.75 - 1.0 \times 5.25 = -6.0000.$$

Composite Coefficients (Table E1)

$$\Gamma_{11} = -0.0249[0.75(4.21) + 1.0 \times 80.125] - 0.9523 \times 0.75 \times 1.432 - 0.1309 \times 1.0 \times 0.75 = -3.3715,$$

$$\Gamma_{12} = 0.6667 \times 6 \times 1.0 - (-3.3715) = 7.3716,$$

$$\Gamma_{13} = -0.75[-0.0249 \times (-13.0) - 0.9523 \times 0.75 \times 1.432] = 0.5240,$$

$$\Gamma_{14} = -0.75 \times (-0.0249) \times 1.0 \times (-6.0) = -0.1122,$$

$$\Gamma_{15} = 0.6667 - [-0.0249 \times 0.0211^2 \times 1.75 - 0.1309] = 0.7976.$$

Welfare Cost Coefficients (Eqs. (E.26)–(E.27))

$$\Omega_y = \frac{7.3716}{0.75^2 \times 1.0^2} = 13.10, \quad \Omega_\pi = \frac{9 \times 0.7976}{2 \times 0.0211} = 169.73.$$

The relatively large value of $\Omega_\pi = 169.73$ reflects the small slope of the New Keynesian Phillips curve ($\kappa = 0.0211$): because prices are highly sticky with a Calvo parameter of $\theta = 0.75$, domestic inflation is costly to generate and hence receives a large weight in the welfare cost function. Conversely, $\Omega_y = 13.10$ reflects the welfare loss from deviations of output from its efficient level, accounting for both the monopolistic competition distortion and the small open economy structure.